In fact... (Part 2/3)

7) The image on the retina would span a micron, the very limit of physical possibility given the wavelength of visible light.

— When you stand on an open plain, how far can you see? [Distance to the horizon $\approx \sqrt{hR} \approx 10^{0.5}$ km, where h = your height, R = radius of the Earth.]

8) 1/6 of the value on the Earth, because $g \propto \rho R$. ρ of the Moon is that of the Earth crust (rock); the Earth core (mostly iron) has a higher density.

— Instead of looking up the value of the Earth-Moon distance D, figure it out from scaling. [Balancing gravity against centrifugal force on the Moon's orbit, $g(R_\oplus/D)^2 = (2\pi D/\text{month})^2/D$.]

9) 4 cm/year. D/(age of the Earth 4.5×10^9 years) gives an over-estimate of 9 cm/year. The tidal braking responsible for the receding was more efficient in the past. Currently the Moon's and the Sun's apparent angular diameters coincide $\approx 1/2°$, which permits total eclipses.

— Estimate the rate $\dot{E}$ of tidal dissipation due to the Moon.

[Let L = Moon's orbital angular momentum, $\dot{\phi}$ = angular speed of its revolution, $\dot{\psi}$ = a.s. of the Earth's rotation. Centrifugal $\dot{\phi}L/D$ = gravity $\propto 1/D^2$, and since $D^2\dot{\phi} \propto L$, we get $D \propto L^2$, $\dot{D}/D = 2\dot{L}/L$. But $L + I\dot{\psi}$ is conserved (I = Earth's moment of inertia), so $\dot{E} = I\ddot{\psi}\dot{\psi} = -\dot{L}\dot{\psi} = -\dot{\psi}/\dot{\phi} \cdot \dot{\phi}L/D \cdot \dot{D}/2 = -\text{month/day} \cdot \text{gravity} \cdot \dot{D}/2$. The gravitational attraction between the Earth and the Moon is 2×10^{20} N, another memorable number. Thus we find $\dot{E} \approx -2.4 \times 10^{20}$ J/year, which is 4 times the annual electricity consumption of the world.]

10) Thickness $\approx k_\text{B}T/mg$, or alternatively $\approx p_\text{atm}/\rho_\text{air}\, g$. Both yield 10 km, the altitude at which jet planes fly.

— How tall a column of water would exert p_atm on the bottom? [About 10 m.]

11) Angle $= 4\,GM/c^2 d$, where d is the impact parameter.

— What variables determine the size of an atom, and how? [Bohr radius $4\pi\varepsilon_0/e^2 \times \hbar^2/m_e \approx \frac{1}{2} \times 10^{-10}$ m, obtainable from dimensional analysis, centrifugal/electrostatic balance, or uncertainty principle.]

12) Drag $\sim \mu\,\ell v$ for Re $\ll 1$, and $\sim \rho\,\ell^2 v^2$ for Re $\gg 1$. In case of a sphere ℓ = radius r, the dimensionless coefficient turns out to be about 6π and 1 respectively.

— What is the speed at which an adult can move through the air so that drag is proportional to speed? [Testing for Re ≈ 1 with $\ell \approx 1$ m, $v = \text{Re} \cdot \mu_\text{air}/(\rho_\text{air}\ell) \approx 2$ microns/sec. Motion on the human scale has drag $\propto v^2$. All those textbook exercises in which it is assumed to incur drag $\propto v$ are bogus.]

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