"Well, Papa, can you multiply triplets?"

Sir William Rowan Hamilton (1805-1865)

Euler's four-square identity

Euler's 4 square identity reads:

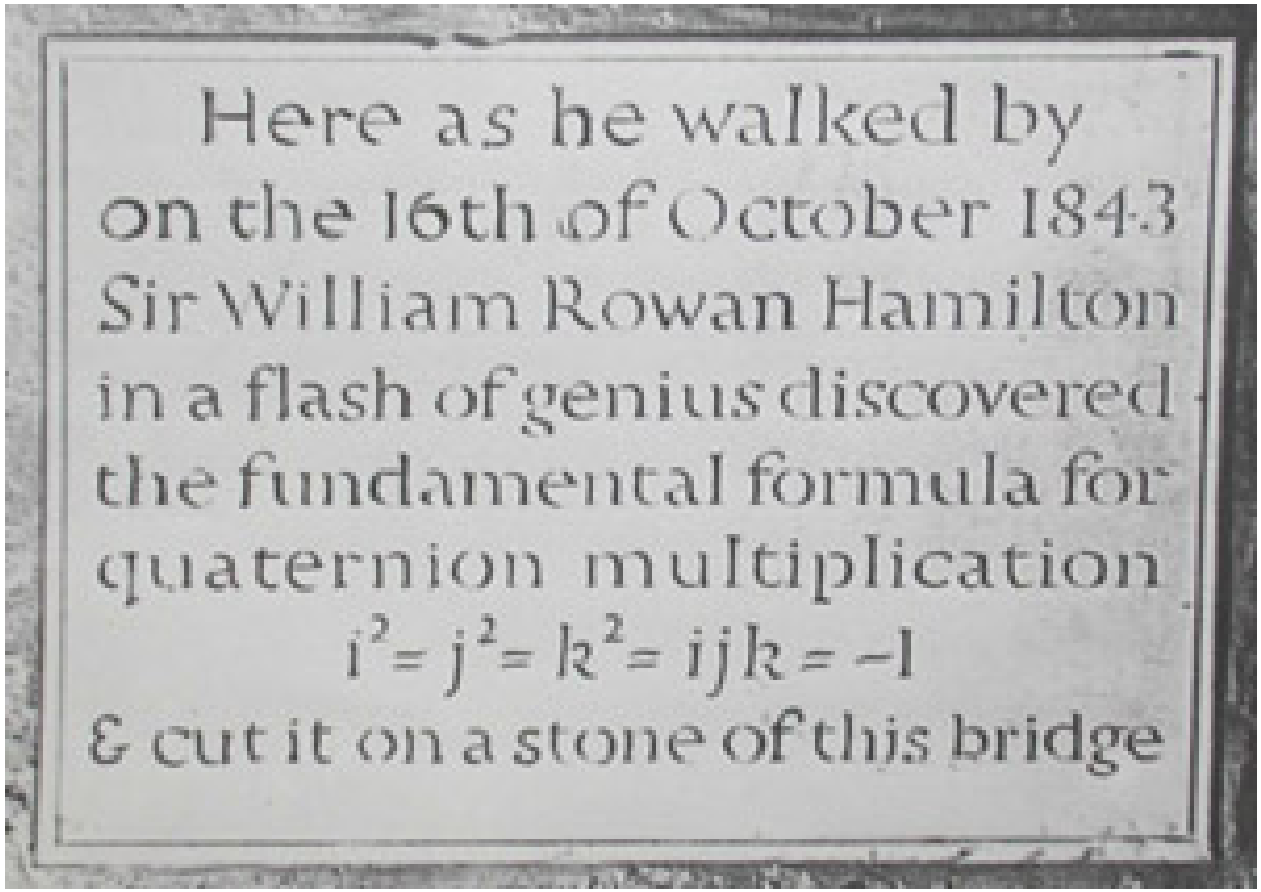
$$\left(a^2+b^2+c^2+d^2\right)\left(e^2+f^2+g^2+h^2\right) =$$

$$\begin{aligned}
&(ae-bf-cg-dh)^2 \\
&+(af+be+ch-dg)^2 \\
&+(ag+ce-bh+df)^2 \\
&+(ah+de+bg-cf)^2
\end{aligned}$$

Degen's eight-square identity

$$(a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2)\times$$
$$(m^2+n^2+o^2+p^2+q^2+r^2+s^2+t^2)=$$
$$(am-bn-co-dp-eq-fr-gs-ht)^2$$
$$+(bm+an+do-cp+fq-er-hs+gt)^2$$
$$+(cm-dn+ao+bp+gq+hr-es-ft)^2$$
$$+(dm+cn-bo+ap+hq-gr+fs-et)^2$$
$$+(em-fn-go-hp+aq+br+cs+dt)^2$$
$$+(fm+en-ho+gp-bq+ar-ds+ct)^2$$
$$+(gm+hn+eo-fp-cq+dr+as-bt)^2$$
$$+(hm-gn+fo+ep-dq-cr+bs+at)^2$$

Discovery of quaternions



The sad truth...