A few formulas

Addition of series :

$$\sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} b_n x^n = \sum_{n=0}^{\infty} (a_n + b_n) x^n.$$

Multiplication of series :

$$\sum_{n=0}^{\infty} a_m x^m \ \cdot \ \sum_{n=0}^{\infty} b_n x^n = \sum_{n=0}^{\infty} c_k x^k \qquad \text{where } c_k = \sum_{i+j=k} a_i b_j.$$

Some examples :

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,$$

$$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} (n+1) x^n,$$

$$(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n,$$

$$\frac{1}{(1-x)^a} = \sum_{n=0}^{\infty} \binom{a+n-1}{n} x^n,$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

$$\frac{1}{\sqrt{1-4x}} = \sum_{n=0}^{\infty} \binom{2n}{n} x^n,$$

$$\frac{1}{\sqrt{1-4x}} \left(\frac{1+\sqrt{1-4x}}{2} \right)^{-\alpha} = \sum_{n=0}^{\infty} \binom{2n+\alpha}{n} x^n.$$

Lagrange's formula :

$$\sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n}{dx^n} \left(f(x)\, a(x)^n \right) = \frac{f(z)}{1 - a'(z)} \qquad \text{where } z = x + a(z).$$

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