In fact... (Part 3/3)

13) A few ($= 10^{0.5}$) mm in diameter. Balance the surface tension $\sigma\, 2\pi(r/2)$ of the neck against the weight $\rho g\, 4\pi r^3/3$ of the bulb as the droplet is about to snap.

— How slowly are water droplets in a cloud falling, and how small are they? [1 cm/sec, for a fog settles down 100 m in hours. Balance viscous drag $6\pi\mu_{\mathrm{air}} r v$ against gravity $\rho_{\mathrm{water}}\, g\, 4\pi r^3/3$ to find $r \approx$ microns. Small enough to scatter light effectively, making clouds look white.]

14) Balance weight $\propto \ell^2$ against drag $\propto \ell^2 v^2$ to get $v \approx$ const. independent of ℓ. This is because paper cones were 2-dimensional: for 3-dimensional solids, weight $\propto \ell^3$ and $v \propto \sqrt{\ell}$.

— How does the swimming speed scale with the swimmer's size? [In a high-Re flow, power consumed $\sim$ drag $\cdot$ speed $\propto \ell^2 v^2 \cdot v$, while power generated $\propto \ell^2$. Balancing, $v \approx$ const. independent of ℓ. In a low-Re flow (e.g. bacteria), drag $\propto \ell v$, which leads to $v \propto \sqrt{\ell}$.]

15) As the animal crouches, rises, and jumps, its muscles exert a force $\propto \ell^2$ over a displacement $\sim \ell$. This accumulated energy $\propto \ell^3$ is converted to potential $mgh \propto \ell^3 h$. Hence $h \approx$ const. independent of ℓ.

— How high *do* real animals jump? [From locusts to humans, spanning a range of mass of 4 orders of magnitude, about half a meter. However, dwarfs like fleas cannot jump that high because, owing to the large area/volume ratio, air resistance literally becomes a drag for them; neither can giants like elephants, who would break their bones if they tried.]

16) Let $v =$ speed of air thrust downward. Thrust mass/time $\propto$ wing area $\cdot\, v \sim \ell^2 v$. The momentum/time $\propto \ell^2 v \cdot v$ must balance the weight $\propto \ell^3$, hence $v \propto \sqrt{\ell}$. The bird's power supplies the kinetic energy/time of the thrust $\propto \ell^2 v \cdot v^2 \propto \ell^{3.5}$. On the other hand, the bird generates power $\propto \ell^2$.

Now draw the graphs of $\ell \mapsto \ell^2$ and of $\ell \mapsto \ell^{3.5}$; coefficients depend only on materials from which all birds are made. They cross at 0 and say ℓ_*. Between 0 and ℓ_*, ℓ^2 is above $\ell^{3.5}$, so the bird has a surplus of power. Beyond ℓ_*, ℓ^2 is below $\ell^{3.5}$, so a deficit.

— How large *is* the largest flying bird? [*Otis tarda* (great bustard), ℓ_* corresponding to 20 kg.]

17) Let $\ell =$ crack length; (energy spent creating the crack) $\propto \ell$, (energy released by the crack created) $\propto \ell^2$, and the critical length corresponds to where the graphs of ℓ and ℓ^2 cross.

— Why at any institution must managers proliferate and end up taking over? [Given N real workers, managers number cN^2, c some coefficient, since managers busy themselves with inter-personal relations, which are modeled as diagonals of an N-gon. As N increases, cN^2 will beat N however small c may be. But there is worse: super-managers will proliferate $\propto N^4$, super-super-managers $\propto N^8$, etc.]

18) At t the walker is likely to be in a volume $\propto \sqrt{t}^{\,d}$, so the probability of her being back at the starting point at this instant decays like $\propto t^{-d/2}$. Recurrence means (Borel-Cantelli lemma) these probabilities get cumulated enough, i.e. $\sum_t t^{-d/2} = \zeta(d/2) = \infty$.

— How does the time to boil an egg scale with its size? [Time $\propto$ diameter^2.]

— In a fluid flow with parameters ρ, v, ℓ, μ, find two time-scales and their physical interpretations.

[$t_{\mathrm{C}} = \ell/v$ and $t_{\mathrm{D}} = \rho\,\ell^2/\mu$, which we call convection time and diffusion time respectively. t_{C} is the time it takes the material fluid to travel around the body. The scaling $\ell = \sqrt{\mu/\rho} \cdot \sqrt{t_{\mathrm{D}}} \propto \sqrt{t_{\mathrm{D}}}$ is characteristic of diffusion: t_{D} is the time it takes for the velocity field to be smoothed out by random spreading, fore and aft of the body. $\mathrm{Re} = t_{\mathrm{D}}/t_{\mathrm{C}}$. Different parts of the flow act as individualists when $\mathrm{Re} \gg 1$, as conformists when $\mathrm{Re} \ll 1$.]

— Why does the accuracy of a guess tend to improve if you decompose it into factors and guess each factor independently ('divide and conquer')? [$g^{\pm 1/n} \cdots g^{\pm 1/n} = g^{(\pm 1 \cdots \pm 1)/n} \approx g^{\pm\sqrt{n}/n} \to 1$ as $n \to \infty$.]

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