Practice in Educated Guessing

Never say 'I don't know.' Come up with *some* guess to every question.

Try to reach the guess from multiple approaches and cross-check. If your estimate is E, you should be confident that the real answer is between $E/10$ and $10E$. Conversely, if you suspect the real answer is between L and U but are unsure of further precision, it is sensible to estimate the answer to be their geometric mean $\sqrt{LU}$, not the arithmetic mean.

The accuracy of a guess tends to improve if you decompose it into factors and guess each factor independently, as you random-walk in the space of exponents.

1) What are the size (length, width, thickness) and the mass of a typical postcard?

2) How many word entries are there in a single-tome dictionary, such as *Pocket OED*?

3) How many primary school teachers are there in France?

4) What is the annual budget of France?

5) How many mathematics papers are published worldwide every year?

6) How much energy is contained in a bowl of rice?

When listing relevant variables, mobilize both your intuition and your scientific education (ideally, they will coalesce at some stage of your life). If you do not know the form of a function $f(x)$, the initial guess should be $f(x) = x$ or $f(x) = 1/x$. You can decide which by thinking whether various variables should be in the numerator or in the denominator.

7) We hear that eagles have such sharp vision that they can spot prey from $\sim 10^3$ m high; 'Bird vision' in `wikipedia` claims that a *Falco sparverius* sees a 2 mm insect from atop an 18 m tree. How reasonable is this?

8) What is the gravitational acceleration on the surface of the Moon?

9) The Moon is steadily receding from the Earth. At what speed?

10) How thick is the atmosphere?

11) According to general relativity, light passing near a star gets deflected. How does the deflection angle scale with various variables?

12) Fluid flows come in two types: creeping (light, slow, small-scale, viscous) or turbulent (heavy, fast, large-scale, not so viscous). Viscosity is represented by μ, with dimension $\mathbf{ML}^{-1}\mathbf{T}^{-1}$, which is the coefficient in

$$\text{shearing force/area} = \mu \cdot \text{ lateral gradient of velocity.}$$

The type of flow is decided by the *Reynolds number* $\mathrm{Re} = \rho v \ell / \mu$. The flow is creeping if $\mathrm{Re} \ll 1$, turbulent if $\mathrm{Re} \gg 1$. When a body of size ℓ is moving at speed v through a fluid of density ρ and viscosity μ, discover how the drag force (fluid resistance) scales with these variables, assuming first that μ is involved (creeping), next that μ is not involved (turbulent).

Nature looks and behaves the way it does as a compromise between competing effects. In any situation, identify the competition.

13) How large is a water droplet that drips from a faucet?

14) Drop paper cones, similarly shaped but of various sizes. How does the terminal velocity scale with the size?

15) How does the jumping height of an animal scale with the animal's size?

16) How does the power required for a bird to fly scale with its size? Show that there exists an upper bound on the size of flying birds.

17) Show that in any brittle material there exists a critical length below which a crack is prevented from growing and beyond which it runs away.

18) It is a key fact of mathematical life that, when we start a random walk, a Brownian motion, any diffusion process, $\ldots$, after time t the probability of presence spreads (standard deviation) to a distance $\propto \sqrt{t}$, independently of the dimension of the ambient space. Explain why the probability that a random walker in a d-dimensional space eventually revisits the starting point is 1 in $d \leq 2$ (recurrence) but is < 1 in $d \geq 3$ (transience).

Tadashi Tokieda, August 2012