The Fibonacci sequence is named after Leonardo of Pisa, who was known as Fibonacci (1175-1240). Fibonacci was born in Pisa, Italy, but he spent a long time in Béjaïa, Algeria, where his father was working as a merchant. He adquired his mathematical education there. He also travelled in Egypt and Syria where he met many mathematicians.

Fibonacci's 1202 book *Liber Abaci* introduced the sequence to Western European mathematics, although the sequence had been described earlier in Indian mathematics, in connection with Sanskrit prosody (Pingala, 200 BC).

The Fibonacci sequence is an increasing sequence F_n, $n = 1, 2, 3, \ldots$ defined by $F_0 = 0, F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$ for $n \geq 2$.

The first elements of the Fibonacci sequence are

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 239, 377, 610, 987, 1597, 2584, 4181, 6765, ...

If

$$\varphi = \frac{1+\sqrt{5}}{2} \sim 1.61803\,39887\cdots$$

and if

$$\overline{\varphi} = \frac{1-\sqrt{5}}{2} \sim -0.61803\,39887\cdots$$

we have

$$F_n = \frac{\varphi^n - \overline{\varphi}^n}{\sqrt{5}}.$$

Johannes Kepler (1571-1630) observed that the ratio of consecutive Fibonacci numbers converges.

He wrote that "as 5 is to 8 so is 8 to 13, practically, and as 8 is to 13, so is 13 to 21 almost", and concluded that the limit approaches the golden ratio φ.

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi.$$

The golden ratio will be our first example of a Pisot number and we have

$$\lambda\varphi^n = F_n + \epsilon_n$$

where $\lambda = \frac{1}{\sqrt{5}}$ and $\epsilon_n = \lambda\overline{\varphi}^n$

The continued fraction representation of φ is given by

$$\varphi = 1 + \cfrac{1}{1 + \frac{1}{1+\frac{1}{1+\dots}}}$$

If we stop this continued fraction at the n-th step, we obtain' the approximation to φ given by

$$1 + \frac{F_{n-1}}{F_n} = \frac{F_{n+1}}{F_n}.$$